Analysis of Fracture Behaviour of UHMWPE GUR1050

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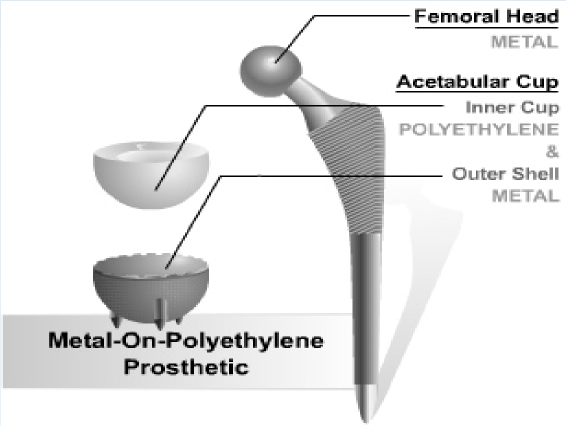
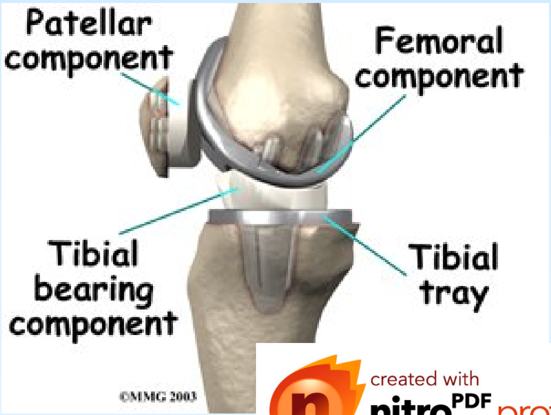
Overview

- Characterise UHMWPE so it can be modelled using computational methods
- Purpose of investigation
- Testing
- Modelling
- Results & Discussion
- Conclusion & Future Research

Purpose

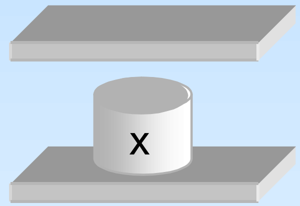
- UHMWPE widely used as a bearing material.
- E.g. hip and knee prostheses.
- Undergoes compressive stresses.
- Most investigations done in tension.
- Explore uniaxial compression behaviour and develop a constitutive model that makes use of Eyring processes. With the potential to generalise to 2 and 3 dimensional behaviour, and use as a basis of the FE analysis of UHMWPE components.





Testing (Materials & Equipment)

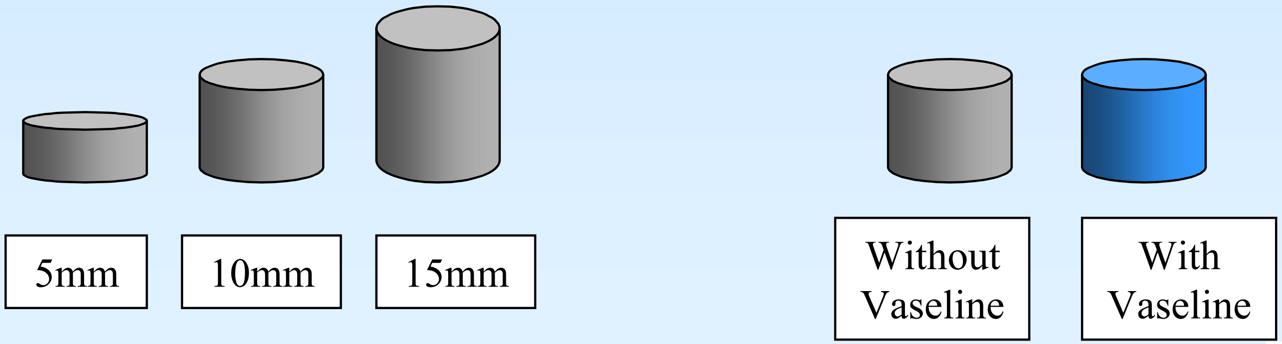
- UHMWPE GUR1050.
- molecular weight $9.2x10^6$ gmol^{-1}, crysallinity between 41.8 and 46.2%.
- Specimens cut on a band saw to create cuboid strips then turned in a centre lathe to the required geometry.
- Geometry was 10mm diameter by 5mm, 10mm, 15mm length and 3.2 μm average surface roughness.



- Uniaxially compressed between steel platens at room temperature on an Instron 5568 Machine.
- Stress-strain and stress relaxation behaviour analysed.
- Temperatures were recorded using an embedded thermocouple to determine adiabatic heating effects.

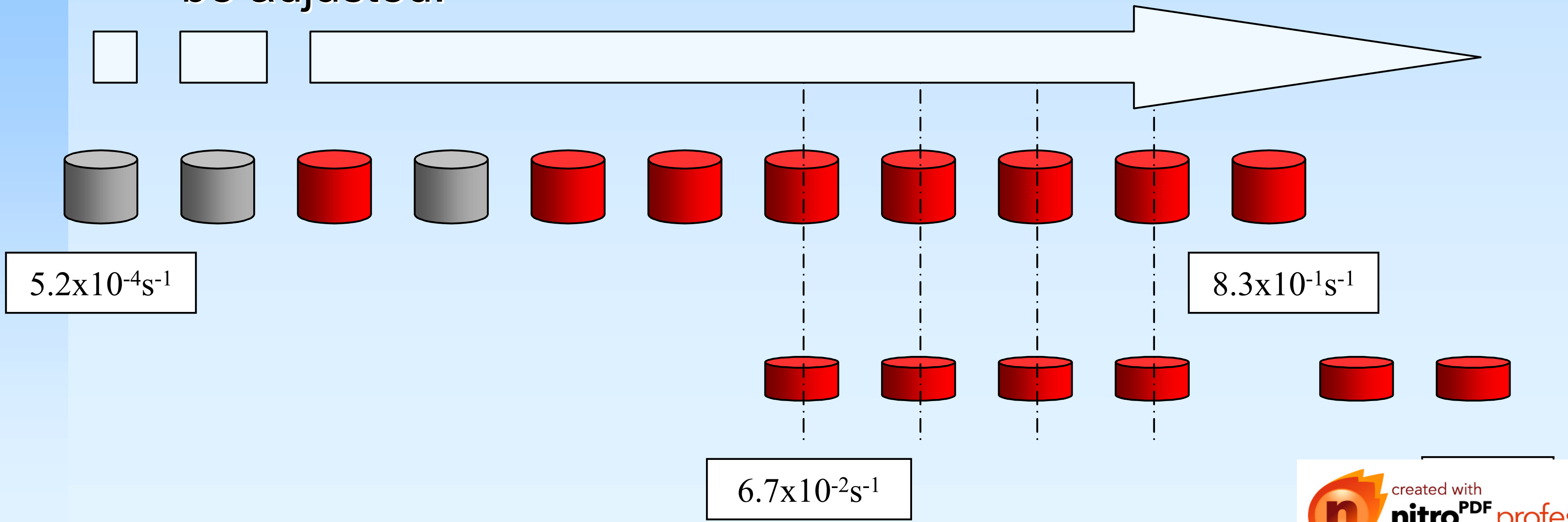
Testing

- Several test sets.
- Extensometer reading compared throughout.
- Set 1.
  - Aspect ratio; 5mm, 10mm, 15mm,20% strain, $1.4x10^{-2}$ s^{-1} strain rates.
  - Vaseline effects; 10mm, 20% strain, 1.7 & $2.5x10^{-2}$ s^{-1} strain rates.



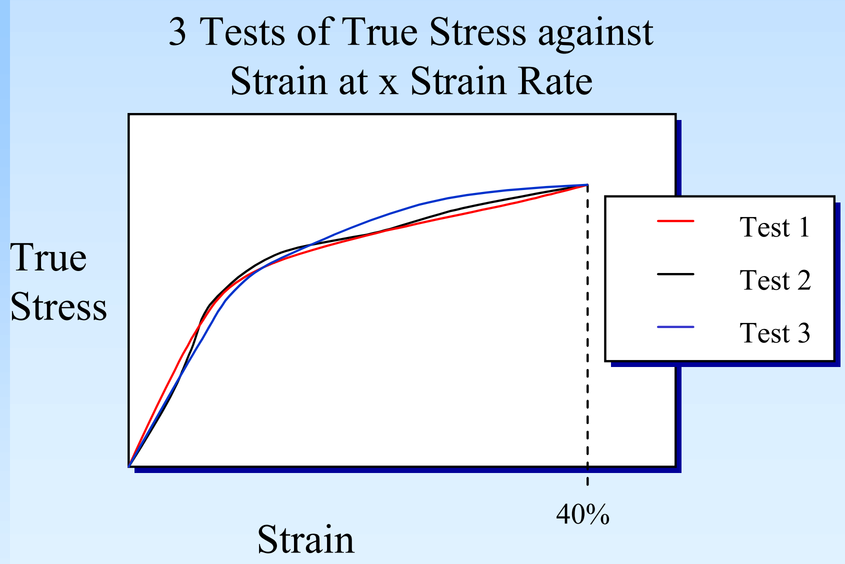
Testing

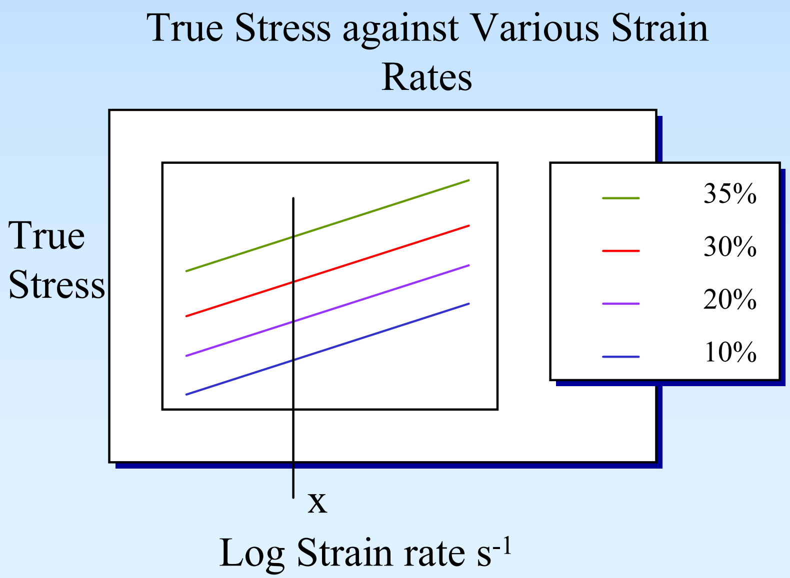
- Set 2.
  - Strain rate tests; 10mm, 40% strain, at rates from $5.2\times10^{-4}\ s^{-1}$ to $8.3\times10^{-1}\ s^{-1}$.
  - Further strain rate tests; 5mm, 40% strain, $6.7\times10^{-2}\ s^{-1}$ to $1.7\ s^{-1}$.
  - Temperature results were recorded so stress values could be adjusted.



Testing
(Range of strains and strain rates)

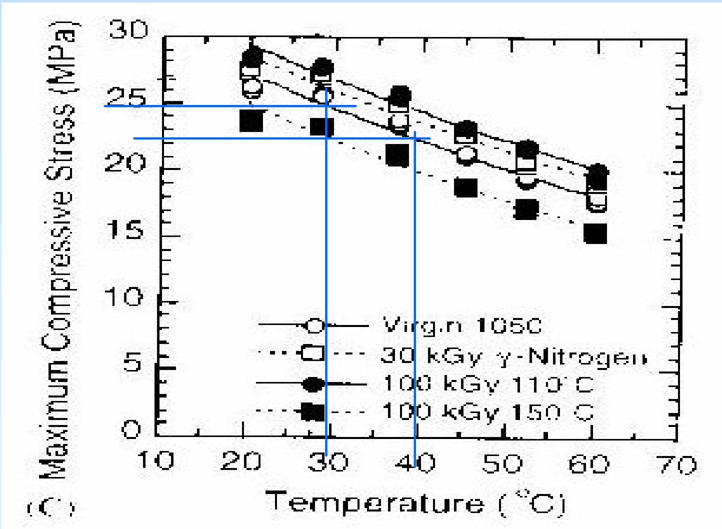
- For the strain rate tests, each specimen compressed to 40% strain but results for 10%, 20%, 30%, and 35% were averaged and displayed graphically



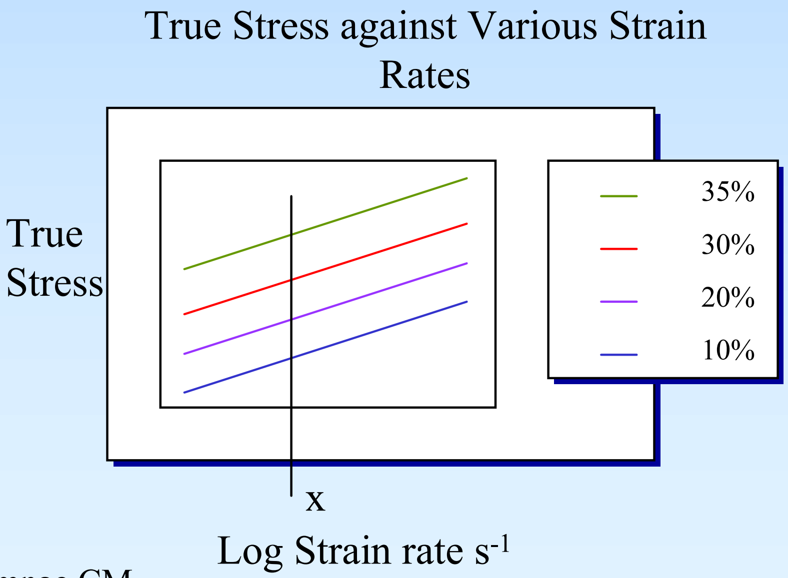


Testing

- Referred to a graph from a paper written by Kurtz and his colleagues; Temperature adjustment; on average 10°C temperature rise corresponded to 2.4MPa drop in stress.
- The stress on the True Stress against Log Strain Rate was adjustment by the corresponding percentage increase.

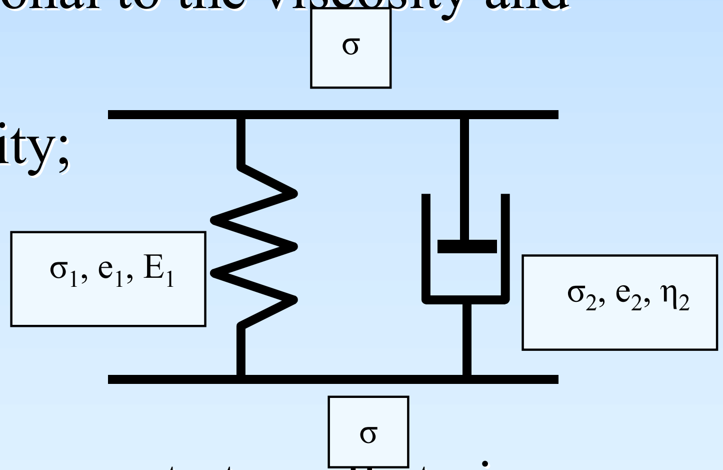


Taken from Kurtz SM, Villarraga ML, Herr MP, Bergstrom JS, Rimnac CM, Edidin AA. Thermomechanical behaviour of virgin and highly crosslinked ultra-high molecular weight polyethylene used in total joint replacements. Biomaterials **23**, 3681-3697 (2002).



Modelling (Linear Viscoelastic)

- Linear viscoelastic modelling
  - Simple
  - Stress is proportional to strain in stress relaxation or creep.
  - Use Hookean springs and Newtonian dashpots filled with viscous fluid e.g. oil to model linear viscoelasicity.
  - Piston in dashpot moves at a rate proportional to the viscosity and applied stress.
- Various models to simulate linear viscoelasicity;
  - The Kelvin or Voigt Model
  - The Maxwell Model
  - The Standard Linear Solid Model



- This proportionality not observed in polymers except at small strains.

Modelling (Non-linear Viscoelastic / Viscoplastic))

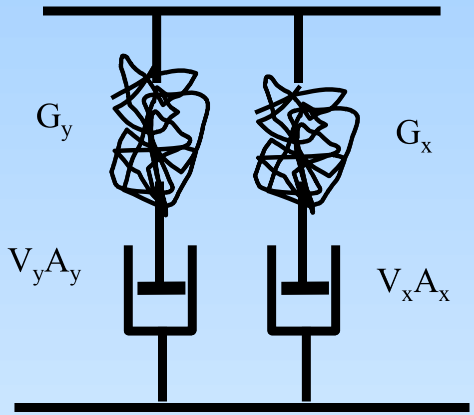
- Integral Models
  - Based on Boltzmann's superposition principle.
- The Eyring Model
  - Assumes deformation of a polymer was a thermally activated rate process
- J_2-plasticity Theory
  - Based on Mises yield surface with an associated flow rule, followed by rate-independent isotropic hardening.

Modelling (Non-linear Viscoelastic / Viscoplastic)

- Arruda-Boyce Model (AB)
  - Elastic and plastic network in series predicts large strain, time- and temperature-dependent response followed by yielding and strain hardening at large deformations.
- Hassan-Boyce Model (HB)
  - similar to AB but activation energy incorporated
- Bergstrom-Boyce Model (BB)
  - gives an equilibrium and time-dependent response
- Hybrid Model (HM)
  - Homogenising the microsstructure into one phase.
  - Internal micromechanical state is tracked as a function of applied deformation
- Augmented Hybrid Model
  - Time dependent viscoplasticity to the backstress network implemented.

Modelling (A constitutive model using Eyring processes developed and implemented numerically)

- A 4 element model with 2 processes was implemented
- Adapted from Wilding and Ward where the elastic elements were changed to Gaussian networks



$$\lambda = \lambda_{GX}\lambda_{EX} = \lambda_{GY}\lambda_{EY} \qquad \sigma_X = G_X\lambda_{GX}^2 - \frac{G_X}{\lambda_{GX}} \qquad \sigma_X = \frac{1}{V_X}\ln\left(\dot{e}/A_X + \sqrt{(\dot{e}/A_X)^2 + 1}\right) \qquad \dot{e}_X = \frac{\dot{\lambda}_{EX}}{\lambda_{EX}}$$

$$\sigma = \sigma_X + \sigma_Y \qquad \sigma_Y = G_Y\lambda_{GY}^2 - \frac{G_Y}{\lambda_{GY}} \qquad \sigma_Y = \frac{1}{V_Y}\ln\left(\dot{e}/A_Y + \sqrt{(\dot{e}/A_Y)^2 + 1}\right) \qquad \dot{e}_Y = \frac{\dot{\lambda}_{EY}}{\lambda_{EY}}$$

λ_G	The extension ratio
V	Activation volume
A	Constant
G	Gaussian modulus

Modelling (Rate-dependent stress-strain behaviour modelled)

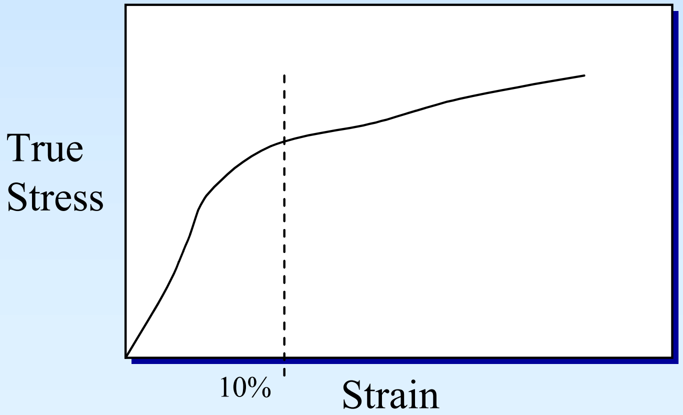
- Previous equations were used to numerically solve for each arm in time increments
- The time derivative $\dot{e}_X = \dfrac{\dot{\lambda}_{EX}}{\lambda_{EX}}$ was first approximated by the finite difference method
- λ_{GX} λ_{EX} found such that $\lambda = \lambda_{GX}\lambda_{EX} = \lambda_{GY}\lambda_{EY}$ applies
- Used Newton's method to produce approximately equal stresses in the stress equations i.e.

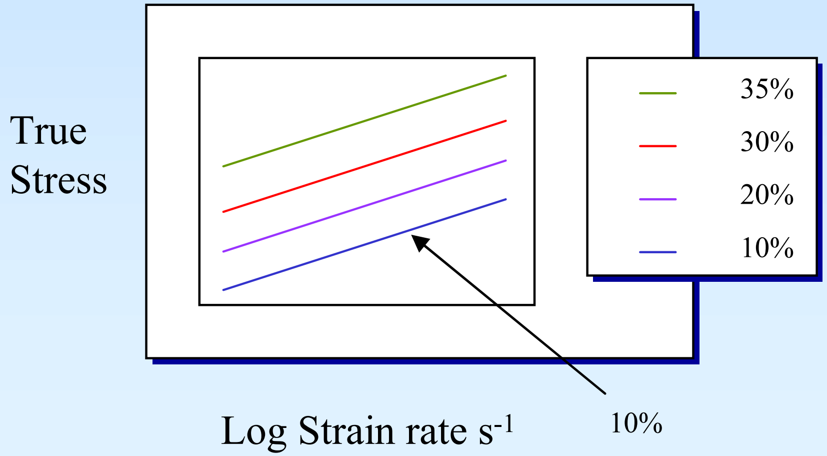
$$\sigma_X = G_X \lambda_{GX}^2 - \frac{G_X}{\lambda_{GX}} \qquad \sigma_X = \frac{1}{V_X} \ln\left(\dot{e}/A_X + \sqrt{(\dot{e}/A_X)^2 + 1} \right)$$

- A similar process applied to the Y and the total stress calculated.

Modelling (6 Parameters derived giving good results)

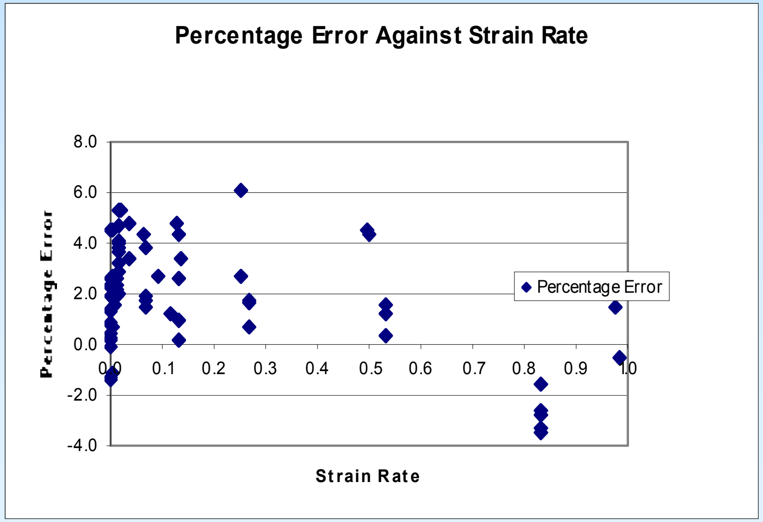
- Stress strain graphs were plotted, gradient of steep part corresponds to the combined effect of G_X and $G_{Y.}$
- The gradient decreases at 10% strain, hence, the gradient of the True Stress against Log Strain Rate at 10% strain was used to derive V_X (volume swept out by a molecular segment during plastic flow).
- V_Y, A_X and A_Y were estimated from observed stress strain curves.

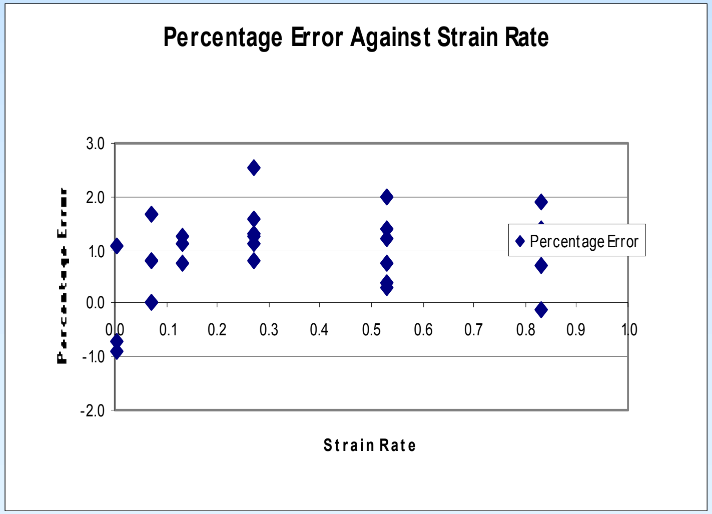




Results & Discussion (Gauge Accuracy)

- The first set of tests show most error lies within 6%
- The second set show most error lies within 3%

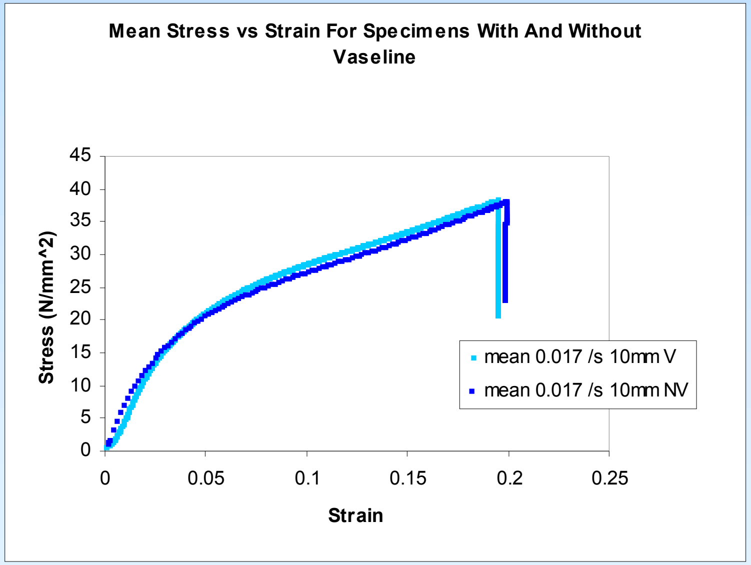




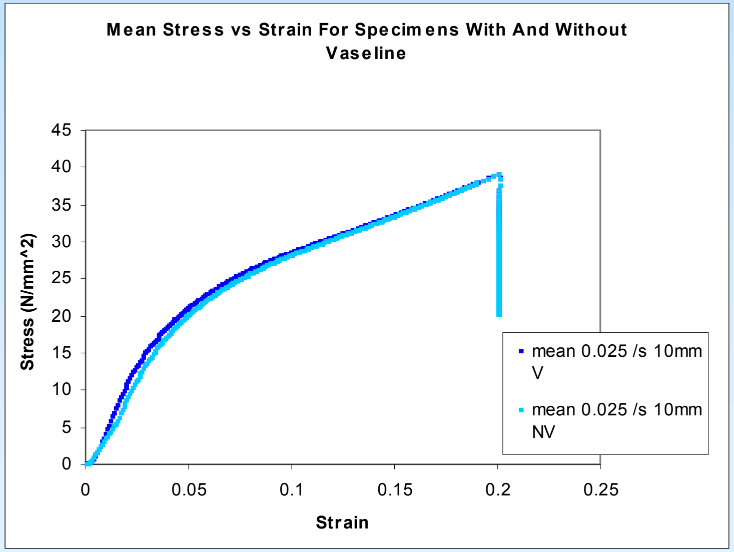
Results & Discussion (Vaseline Lubricant)

- No significant difference at either strain rate; $1.7x10^{-2}$ s^{-1} or $2.5x10^{-2}$ s^{-1}

$1.7x10^{-2}$ s^{-1}

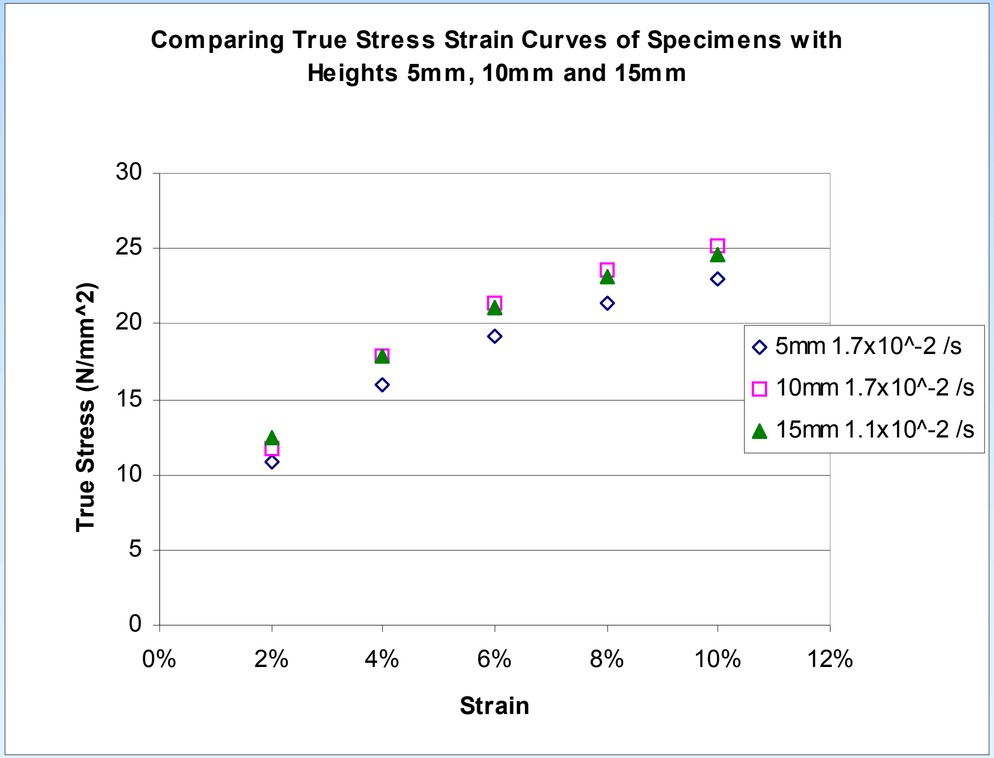


$2.5x10^{-2}$ s^{-1}



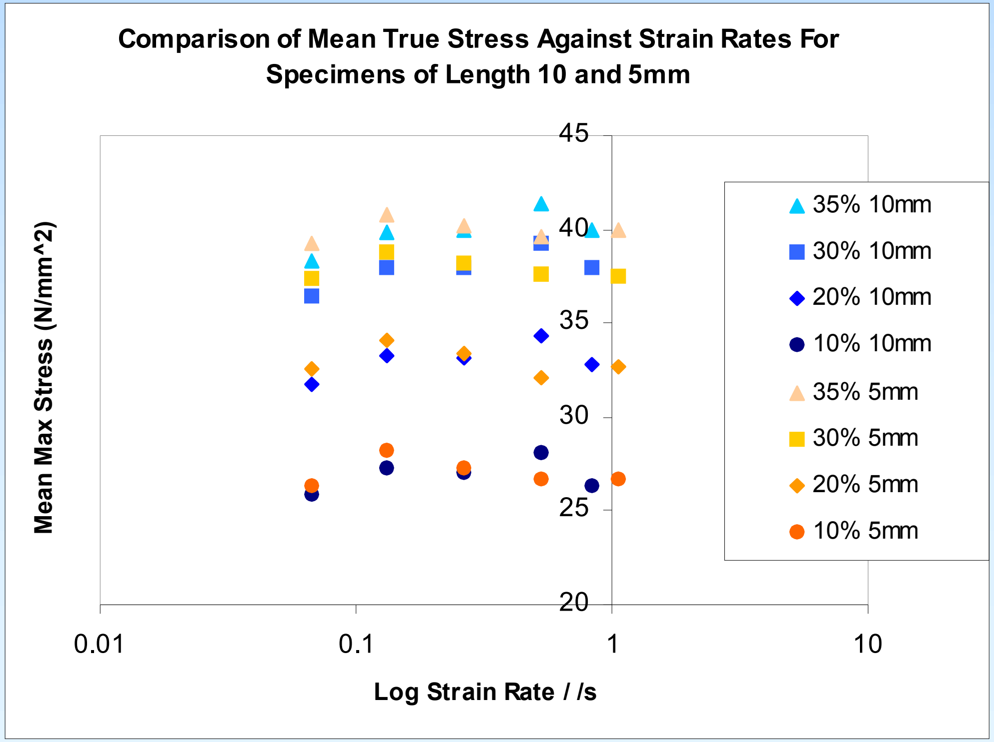
Results & Discussion (Aspect Ratio)

- Aspect ratio affects results; taller specimens experience greater stress.



Results & Discussion (10mm / 5mm)

- The 10mm length specimens seem to experience a linear increase which dips at the higher strain rates.
- The 5mm long specimens experience a dip at 5.3×10^{-1} s^{-1} affecting the results, otherwise similar results.



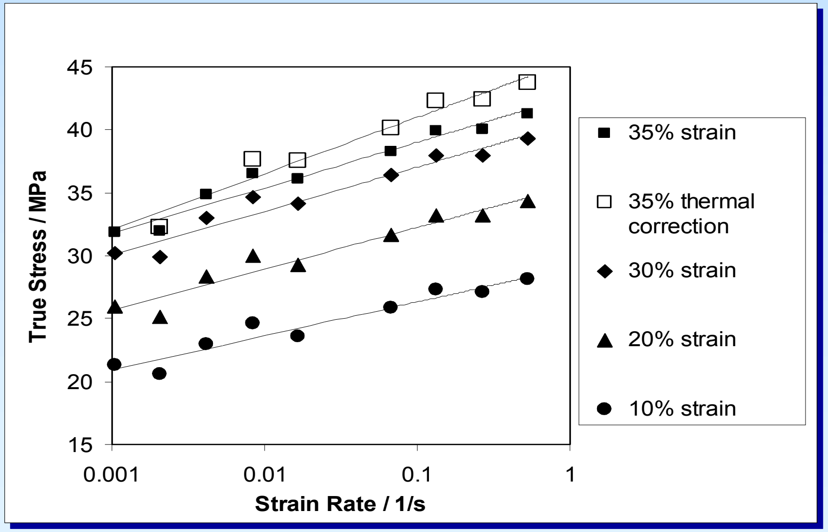
Results & Discussion (Parameters / Heating)

Table 1

Arm	Parameter	G /MPa	A / s^{-1}	V / MPa^{-1}
X		120	1.4×10^{-12}	1.3
Y		15.5	4.8×10^{-15}	0.94

- Adiabatic heating is affecting the results, greatest at higher strain rates
- Not significant in the evaluation of our constitutive model

Figure 1 Rate dependence



Results & Discussion (Modelling)

- Results show that model gives a good representation over strain rates ranging from 5.2×10^{-4} to 5.3×10^{1} s^{-1}
- 2 processes and Gaussian network simplify the model and cause the abrupt gradient transition
- Non-uniaxial data are required to fully assess the model

Figure 3 Stress-strain behaviour at rate 5.2×10^{-4} s^{-1}.

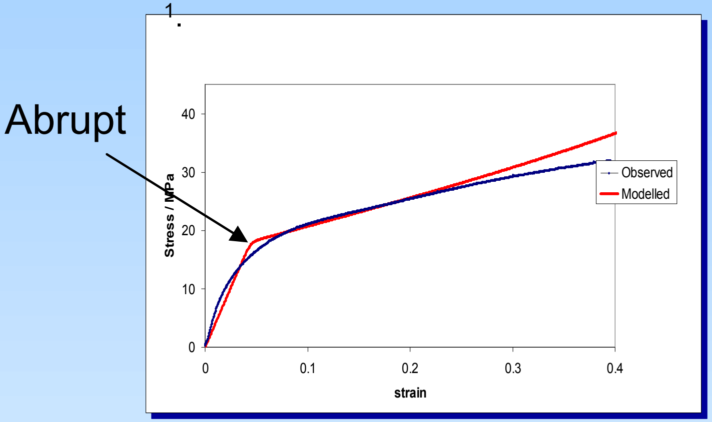
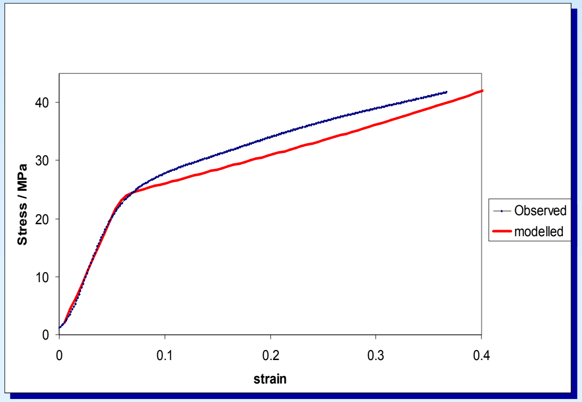


Figure 4 Stress-strain behaviour at rate 0.53 s^{-1}.



Conclusion & Further Research

- Results are encouraging
- The advantage of this model is it can represent all qualitative features of polymer behaviour
  - Stress relaxation
  - Creep
  - Recovery of strain and stress
- This model offers greater computational efficiency compared with the hereditary integral approach
- Further verification in process
- Implementing this model into FE code

Thank you for your attention

Hereditary / Single Integral Approach

- Boltzmann superposition approach assumed creep is a function of past loading history and the final deformation can be calculated by summing past loading steps

$$e(t) = \int_{-\infty}^{t} J(t-\tau)\,d\sigma(t)$$

- Leaderman's theory replaced stress by a function of stress *f(σ)*

$$e(t) = \int_{-\infty}^{t} \frac{df(\sigma)}{d\tau} J(t-\tau)\,d\tau$$

- Pipkin and Rogers theory defined a non-linear creep function C

$$e(t) = \int_{-\infty}^{t} \frac{d\sigma}{d\tau}(\tau)\, C(t-\tau,\sigma(\tau))\,d\tau$$

- Schapery theory replaces stress by a function of stress and time by a function of time using Leaderman's model

$$e(t) = g_0 D_0 \sigma + g_1 \int_{0}^{t} \Delta D(\varphi - \varphi') \frac{dg_2\sigma}{d\tau}\,d\tau$$

Comparison Of Single Integral Models

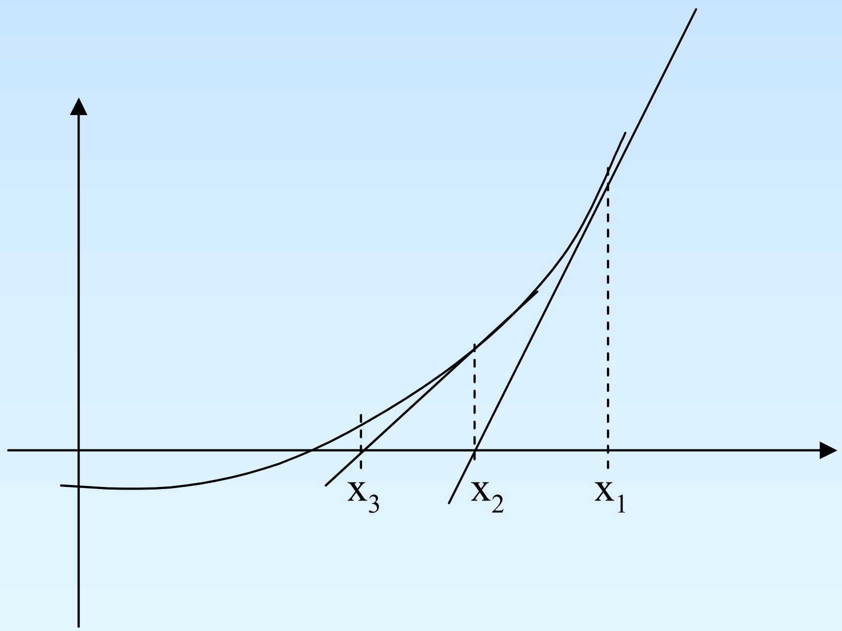
- Smart and Williams compared the Pipkin and Rogers, Schapery theory and the BKZ models
- Schapery performed the best when applied to polypropylene and poly(vinyl chloride) fibres under tensile strains up to 4%.

Finite Difference Method

- Identify the problem
- Create differential equations
- Solve the equation approximately

Newtons Method

- A point is taken x_1 and the gradient evaluated to find the intercept on the x axis.
- This is point x_2 and the gradient is again evaluated to find the corresponding intercept x_3
- The process is repeated to solve the equation.



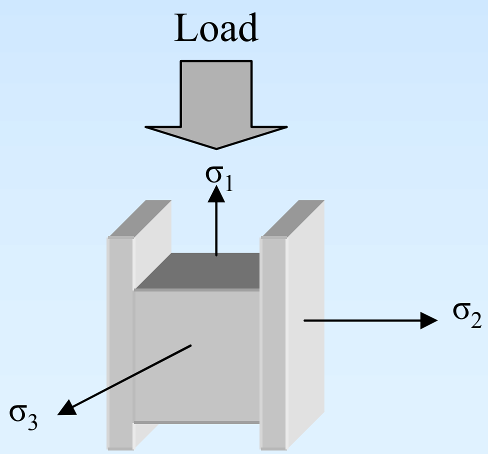
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Other Applications

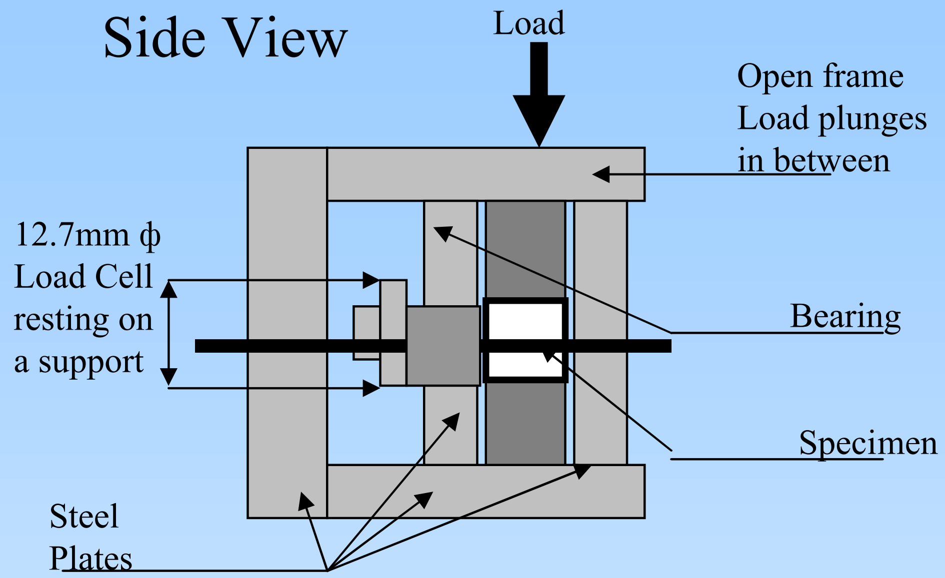
- Applicable to other semi-crystaline polymers as likely to follow this 2 process model
- Paul Buckly used the 1 process to model PMMA
- John Sweeney using to model polycarbonate with good results

Multi-axial Testing

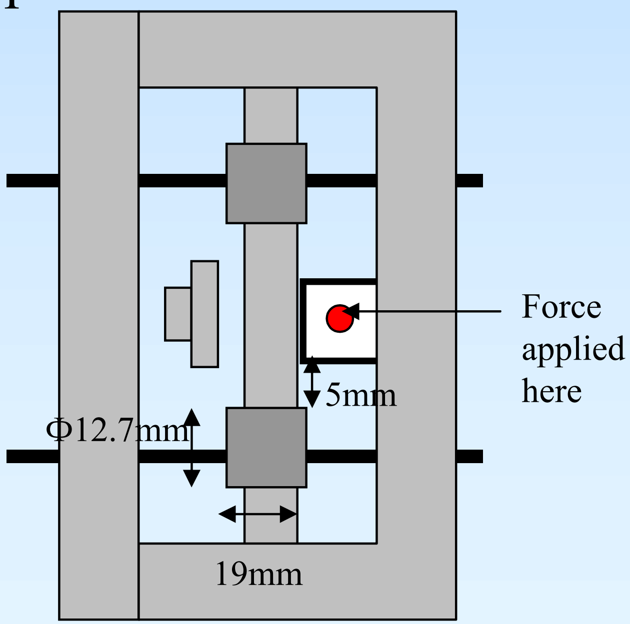
- Plane strain compression where strain e = 0 in cuboid sections.
- Restrain sections from opposite sides and compress at top so 3 different stresses exist.



Side View

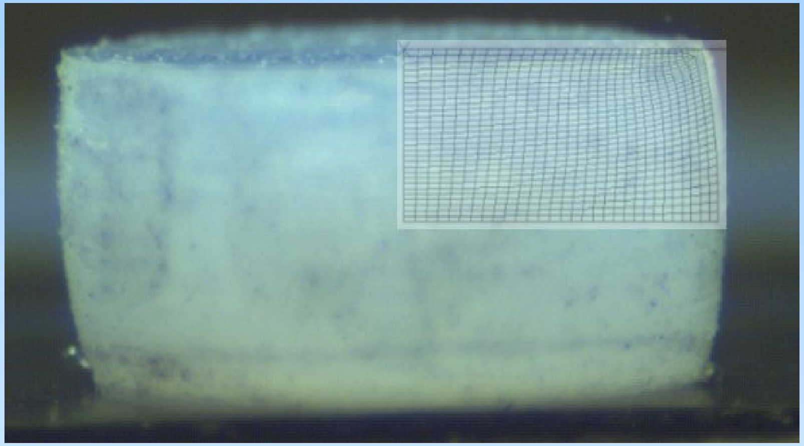


Top View

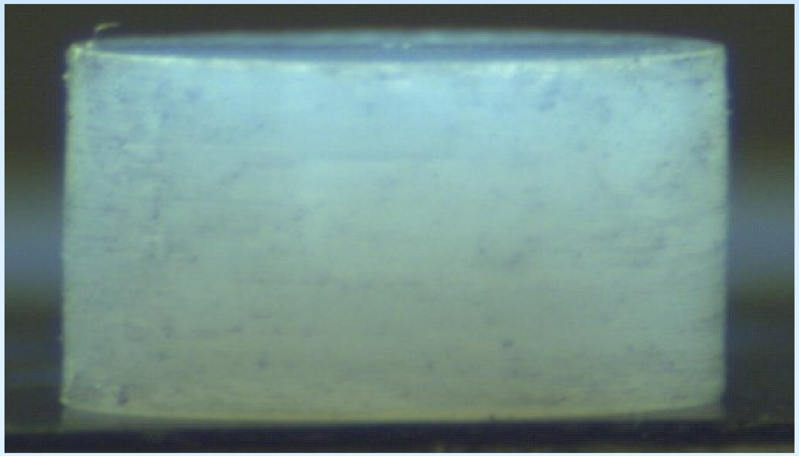


Abaqus model against actual test

- 0.1 friction coefficient in Abaqus
- 1.667 s^{-1} strain rate (500mm/min max speed) in actual test

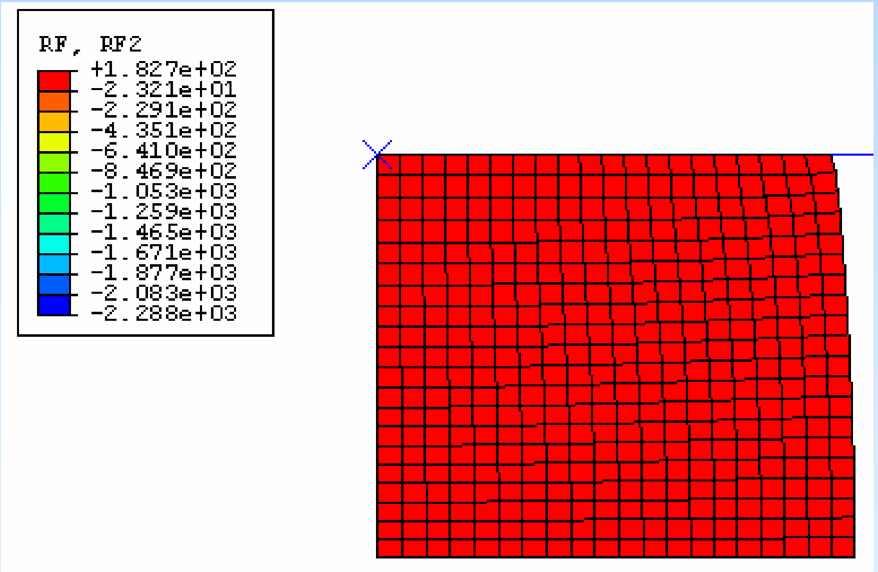


- 0.83 s^{-1} strain rate (250 mm/min speed)

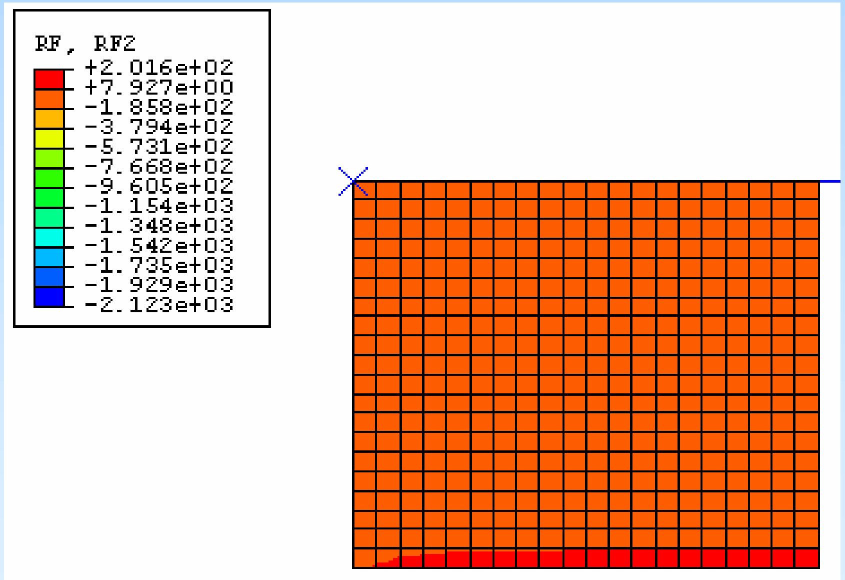


Reaction Forces Using FEM

Reaction force when friction is 0.4



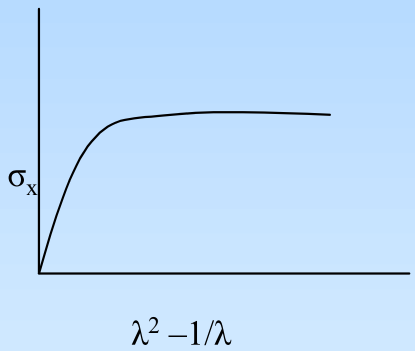
Reaction force when friction is 0



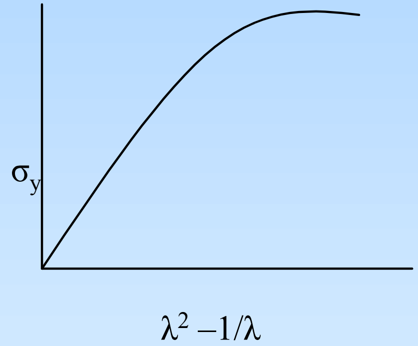
Calculating Parameters G_x and G_y

- $\sigma_X = G_X\,(\lambda^2\text{-}1/\lambda)$ so plotting σ against $(\lambda^2\text{-}1/\lambda)$ we get the gradient which is equivalent to G_X.

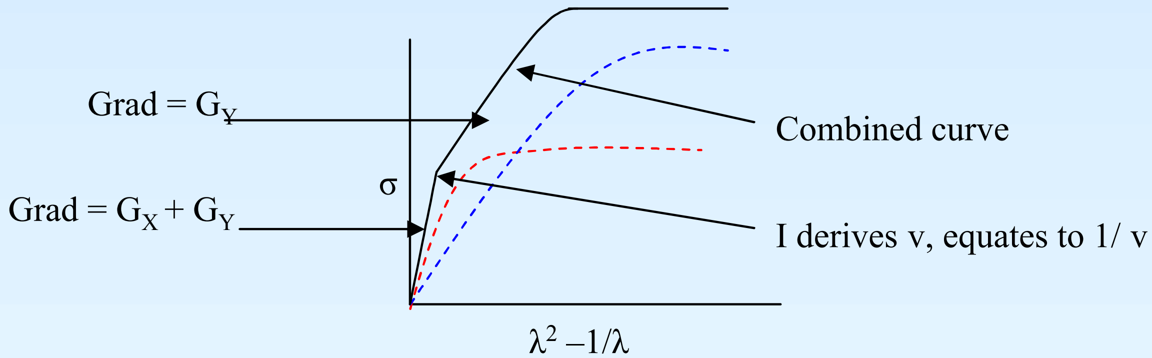
σ_x against $(\lambda^2\text{-}1/\lambda)$



σ_y against $(\lambda^2\text{-}1/\lambda)$



Total σ against $(\lambda^2\text{-}1/\lambda)$



Calculating V_x (Activation Volume including Shear and Hydrostatic Stress)

$$\dot{e} = A \sinh V\sigma$$

where strain rate $\dot{e}$ produced as a result of uniaxial stress σ acting on an eyring process follows the above equation and A is a constant. Strain rate approximates to;

$$\dot{e} \approx \frac{1}{2} A \exp(V\sigma)$$

Rearranging we have;

$$\frac{2\dot{e}}{A} = \exp(V\sigma)$$

Taking natural logs we arrive at

$$\ln\left(\frac{2\dot{e}}{A}\right) = V\sigma \ln e$$

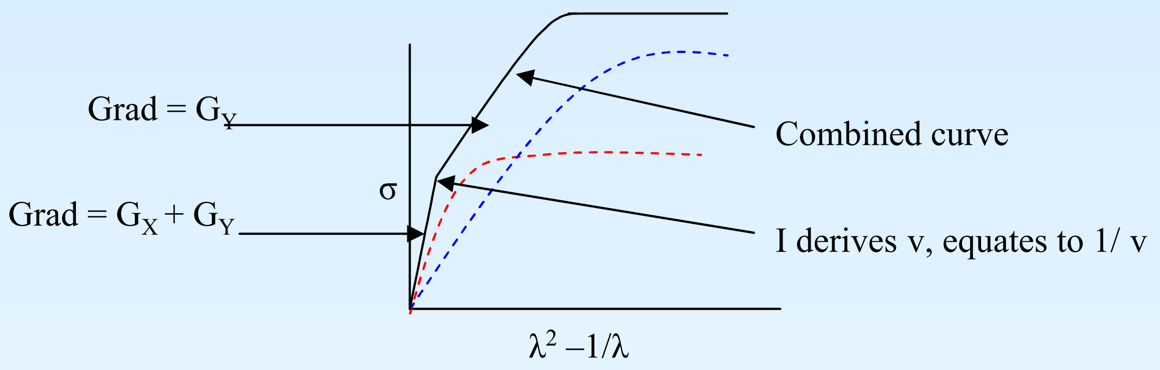
So stress is $\sigma = \frac{1}{V}\ln\left(\frac{2\dot{e}}{A}\right)$ Or $\sigma = \frac{1}{V}\left(\ln\dot{e} + \ln\left(\frac{2}{A}\right)\right)$

Calculating V_x (Activation Volume including Shear and Hydrostatic Stress)

This shows that if we plot stress at the inflexion point, (I as indicated on above graph) against strain rate, the gradient will be 1/V.

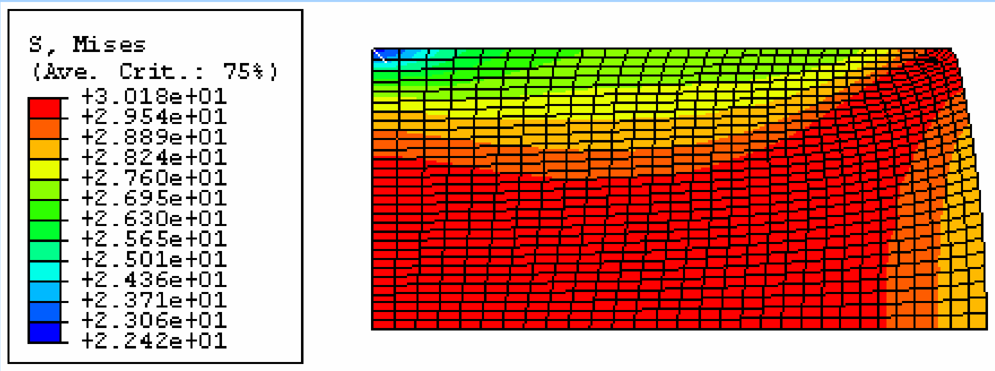
Fig 1 Rate Dependence graph in the report 'Viscoplastic Behaviour of Ultra-High Molecular Weight Polyethylene' written by Naz et al. is such a graph and was used to evaluate V_X.

Total σ against (λ^2-1/λ)



Thermocouple Position

Coefficient of friction 0.1, stress is at centre region.



Coefficient of friction 0, stress is uniform.